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## TECHNICAL NOTE

### SEGMENT INERTIAL PARAMETER EVALUATION IN TWO ANTHROPOMETRIC MODELS BY APPLICATION OF A DYNAMIC LINKED SEGMENT MODEL

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**Abstract**—The estimation of segment inertial parameters (SIPs) is an important source of error in inverse dynamic analysis. In most individual cases SIPs are derived from extrapolation of known SIPs of a certain population through regression equations (proportional models). Another well-known method is the use of mathematical approximation of the shape of human body segments combined with estimations of segment densities (geometric models).

In the current study five males and five females performed four different lifting movements in the sagittal plane. A full body linked segment model was applied twice to the same data set, once using a proportional and once using a geometric anthropometric model. As a full body linked segment model is an overdetermined system of equations, four equations could be formed to test the summed effect of SIP errors on the inverse dynamic analysis. The overall performance in terms of coefficients of correlation was better for the geometric model as compared to the proportional model. When a back lifting movement was performed, the equations indicated systematic errors in the proportional model. However, when a leg lifting movement was performed, the equations indicated systematic errors in the geometric model. Therefore, analyzing only one kind of movement does not suffice to draw conclusions with respect to the reliability of an anthropometric model.

#### INTRODUCTION

Inverse dynamic analysis can be used to calculate the bio-mechanical load on human body segments (Elftman, 1939). The analysis requires a model in which the human body is represented as a chain of rigid segments. These segments are interconnected by joints, which are (mostly) considered as hinge joints. Newtonian mechanics are applied to each segment to calculate net joint moments and forces. The reliability of the model output depends on assumptions like the rigidity of human body segments and on the degree of accuracy of the kinematic and anthropometric data. Some possible sources of error are: the estimation of joint rotation centers (Looze *et al.*, 1992b); varying segment lengths, especially observed in the trunk (Looze *et al.*, 1992a); skin movement artifacts (Capozzo *et al.*, 1993); errors in measuring forces and marker positions. Another main source of errors is the estimation of segment inertial parameters (SIPs). According to Capozzo and Berme (1990), the magnitude of SIP errors can be up to 48, 25 and 80% for the segment mass, relative center of gravity and frontal axis moment of inertia, respectively. In order to obtain reliable output of the inverse dynamic analysis it is important to reduce these errors.

#### Measurement of segment inertial parameters

The best method to determine SIPs is of course to measure them. *In vivo* assessment of SIPs is possible though difficult. The

gamma scan method (Zatsiorsky and Seluyanov, 1985; Zatsiorsky *et al.*, 1990) is a combination of volume and density measurements from which SIPs can be calculated. However, this method is expensive and exposes subjects to radiation. Another way of *in vivo* assessment of SIPs is volume measurement by means of submersion (Plagenhoef, 1983) or photogrammetric methods (McConville *et al.*, 1980). This method requires assumptions about segment density. These methods have in common that they are complex and time consuming. Therefore, they are not widely used in routine biomechanical analysis (Capozzo and Berme, 1990). The methods that are most often used estimate SIPs. They can be divided in two methodological groups, proportional and geometric models, which will be described below.

#### Proportional anthropometric models

Proportional anthropometric models estimate SIPs by regression equations, requiring, respectively, one or more relevant anthropometric measures (e.g. total body mass) as input. One problem of these models is that they are based on SIP measurements of rather homogeneous populations. Some of the proportional models are based on a small sample of cadaver SIP measurements in relatively old males, e.g. 8 males of  $68.5 \pm 11.0$  yr by Dempster (1955) and 13 males of  $49.3 \pm 3.8$  yr by Clauser *et al.* (1969). Others are based on *in vivo* assessment of SIPs of a larger but quite specific population, e.g. physical education students (Zatsiorsky *et al.*, 1990), athletes (Plagenhoef, 1983) or soldiers (McConville *et al.*, 1980). While the SIP errors in proportional models might be relatively small within the populations the studies are based on, the errors and the uncertainty about their magnitude will grow when the models are applied to subjects with anthropometric characteristics differing from the mean of that population.

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### Geometric anthropometric models

In geometric anthropometric models the SIPs are calculated from simple geometric representations of body segments. Some examples of geometric representations are ellipsoids and elliptical cylinders, which were used by Hanavan (1960) or segments divided in small elliptical zones (Jensen, 1986). A very complex model, consisting of a variety of geometric forms, was built by Hatze (1980). The drawback of this method is that it requires 248 measurements on each subject as input. Recently, Yeadon (1990a) suggested the division segments into subsegments, bounded by transverse planes, consisting of a rectangle with a semi-circle on each side. Yeadon stated that these shapes are closer to the real shape of the human body than ellipses, especially in the trunk. Geometric models of human segments have the advantage that they can (in principle) be used for any population. The only assumptions to be made are assumptions on segment densities. However, errors can be introduced by oversimplification of segment shapes. A more extensive review of the methods described (except for the method of Yeadon, 1990a), is given by Capozzo and Berme (1990).

### The effect of segment inertial parameter errors in inverse dynamic analysis

The question arises whether the use of proportional and geometric models leads to systematic differences in SIPs. If it does, which model gives the most reliable estimates of net moments and forces in the inverse dynamic analysis?

In a full body linked segment model with known external forces, the system of equations is overdetermined (Vaughan, 1982). This suggests that there is some information available to test the reliability of estimated net moments and forces. Indeed, some methods can be found in the literature, for instance comparison between measured and estimated ground reaction force (Kromodihardjo and Mital, 1986) or between the results from a top-down and a bottom-up net moment calculation (MacKinnon and Winter, 1993; Looze *et al.*, 1992b).

In the present study 10 subjects with varying anthropometric characteristics performed 4 simple lifting movements in the sagittal plane. The comparisons between measured and estimated ground reaction force and between top-down and bottom-up calculated net moments were used to study the reliability of estimated net forces and moments. Additionally, two more comparisons were added to test the quality of the model output. The first is a comparison between the horizontal position of the body center of gravity and the center of pressure. The second is a comparison between the external moment and the rate of change of the angular momentum.

The inverse dynamic analysis was applied for both a proportional and a geometric anthropometric model. The proportional model was based on a model developed by Plagenhoef (1983) from a combination of cadaver data from Dempster (1955) and submersion of segments of a group of young males and females.

The second model that was applied, was a geometric model, as suggested by Yeadon (1990a). The aim of this study was to find out which of the two models gives the most reliable results in an inverse dynamic analysis.

## METHODS

### Subjects

Five male and five female subjects participated in the experiments. In order to prevent systematic errors caused by selecting a homogeneous group of subjects, care was taken to select subjects with widely ranging anthropometric characteristics. Subject characteristics are presented in Table 1. All subjects signed an informed consent prior to the experiments.

### Procedure

The subjects were asked to lower and lift a barbell of 10.4 kg from knuckle height to just above the ground and back with a fixed speed using prescribed lifting techniques. After some practice the subjects performed four lowering/lifting cycles differing in technique and speed. The techniques used were a back technique where the knees are straight and the torso is flexed and a leg technique where the torso is straight and the knees are flexed. The subjects started each task in an upright standing position with the barbell in their hands. Both techniques were performed at two different speeds. After lowering the barbell in 1 s (fast) or 4 s (slow) the subjects waited for 1 s before they lifted the barbell in 1 s (fast) or 4 s (slow).

### Proportional anthropometric model

The total body mass and the segment lengths of each subject were measured. SIPs were calculated as follows: segment mass is a ratio of the body mass; radius of gyration (from which the segment moment of inertia is calculated) is a ratio of the segment length; the position of the segment center of gravity is a ratio of the line between 2 joint centers indicated by markers. For all three SIPs, ratios described by Plagenhoef (1983) for men and women were used. Data from Liu *et al.* (1971) on the SIPs of lumbar vertebral segments were scaled to subject length and used to recalculate the trunk and pelvis data from Plagenhoef such that a segmentation plane between the pelvis and the trunk at the L5-S1 joint was obtained.

### Geometric anthropometric model

Each segment is divided into a number of solids (1 for the feet, 2 for the pelvis, 3 for the hands and head plus neck, 4 for the upper arms and forearms, 5 for the upper legs, 6 for the trunk and 7 for the lower legs), each bounded by two parallel stadia. At each stadium the height, perimeter (*p*) and width (*w*) were measured. Yeadon (1990a) defined a stadium as a rectangle of width

Table 1. Anthropometric characteristics of the subjects participating in the experiment

Males				Females			
Subj.	Age (yr)	Body mass (kg)	Stature (m)	Subj.	Age (yr)	Body mass (kg)	Stature (m)
M1	18	64.7	1.77	F1	25	67.2	1.64
M2	30	64.8	1.85	F2	21	52.3	1.66
M3	23	79.4	1.93	F3	44	50.2	1.58
M4	20	83.8	1.83	F4	21	68.0	1.68
M5	26	89.5	1.79	F5	20	70.7	1.77
Mean	23	76.4	1.83	Mean	26.2	61.7	1.67
S.D.	4.8	11.3	0.06	S.D.	10	9.6	0.07

2t and depth 2r with an adjoining semi-circle of radius r at each end of its width. r and t are calculated as

$$r = (p - 2w)/(2\pi - 4) \quad \text{and} \quad t = (\pi w - p)/(2\pi - 4).$$

For each solid a volume was calculated according to Yeadon (1990a). An estimated body mass was calculated using the segment density values of Dempster (1955). However, in this way the total body mass was overestimated by  $6.0 \pm 2.9\%$ . Therefore, the density values are scaled to the ratio of measured body mass divided by the estimated body mass. The equations used to calculate volume, vertical position of the center of gravity and the moment of inertia of solids were derived from Yeadon (1990a) and are described in Appendix A. The inertia parameters of all solids that form a segment were summed to obtain SIPs.

Additional measurements of the distance between the front side of the body and a marker position at the joint centers enabled calculation of the mid-points of the joint stadia on the sagittal axis during the lifting cycles. Calculation of these mid-points is necessary since the model defines the segment center of gravity on a central longitudinal line in the segment and not (as the proportional model) on a line connecting joint centers.

#### Kinematics and kinetics

Reflective markers were attached to landmarks at the fifth metatarsal joint, the ankle joint at the distal part of the lateral malleolus, the knee joint at the lateral epicondyle, the hip joint at the greater trochanter, the lumbo-sacral (L5-S1) joint (according to Looze *et al.*, 1992b), the spinous process of the first thoracic vertebra, caput mandibula at the head, the lateral border of the acromion, the elbow joint at the lateral epicondyle, the wrist joint at the ulnar styloid, and a small stick attached to the third metacarpus.

During the lifting cycles marker positions were recorded at 60 Hz using a three-dimensional automatic video-based motion registration system (VICON, Oxford Metrics). Ground-reaction forces were recorded simultaneously by a force-platform (Kistler, 9218B).

Sagittal marker coordinates and analog to digital converted force signals were digitally filtered by a fourth-order Butterworth filter with zero phase lag at an effective cut-off frequency of 5 Hz. Segment angles were obtained as the angle between the line connecting two successive markers and the right horizontal (for the trunk the L5-S1 marker and the thoracic marker were used). Joint positions were represented by markers, except for the shoulder joint where the joint was calculated on the line acromion-lateral epicondyle.

Centers of gravity were calculated as a ratio of the distance between two successive markers (except for the hands-barbell and the trunk segment) for the proportional model. In the geometric model, first, the mid-points at the sagittal axis of all stadia at the end of the segments were calculated using marker positions and the measured distances to the front side of the body. Then the segment centers of gravity were determined as a ratio between the two calculated sagittal mid-points.

For both models trunk angle and trunk center of gravity were tracked during the movement using the L5-S1 and thoracic marker (according to Looze *et al.*, 1992b).

Segment linear and angular accelerations were obtained from the time histories of, respectively, the segment center of gravity positions and the segment angles by double differentiation with a Lanczos 5-point numerical differentiator.

#### Linked segment model

The linked segment model applied was a dynamic two-dimensional linked segment model, described by Looze *et al.* (1992b). In short: 9 segments are defined in a sagittal plane: feet, lower legs, upper legs, pelvis, trunk, head, upper arms, forearms and hands/barbell. The definition of a separate head segment is a small modification to the Looze *et al.* model. The segments were assumed to be rigid and connected to each other by intersegmental hinge joints. Net moments and forces were cal-

culated by means of an inverse dynamic analysis (Elftman, 1939). The linked segment model was applied to the same kinematic and kinetic input set twice: once the SIPs were derived from the proportional model and once they were derived from the geometrical model.

#### Error analysis

Since the system of equations in a linked segment model is over-determined, there are some possibilities to test the reliability of the results. For this purpose four equations will be used to produce relevant error terms.

The first equation is based on the mechanical equality between the ground reaction force vector ( $F_{gr}$ ) plus the bodyweight vector ( $m \cdot g$ ;  $g$  is negative), on the one hand, and the summed product of segment masses ( $m$ ) and their linear acceleration vectors ( $a$ ), on the other hand:

$$F_{gr} + m_b g = \sum_{j=1}^n (m_j a_j) + \varepsilon_1. \quad (1)$$

In equation (1) the subscript  $j$  indicates the  $j$ th segment and  $n$  is the total number of segments. The error term in this equation is indicated by  $\varepsilon_1$ . The SIPs that can influence  $\varepsilon_1$  are the segment mass and to a small extent the segment center of gravity. The position of the center of gravity with respect to the joint rotation axis influences the calculated linear acceleration due to rotational movements.

The inverse dynamic analysis was performed twice for both models: once starting at the hands and once at the feet. Consequently, a second equation can be described where net moments calculated in the analysis starting at the feet ( $M_f$ ) should equal the net moments calculated in the analysis starting at the hands ( $M_h$ ). Thus, for each joint:

$$M_{fj} = M_{hj} + \varepsilon_2 \quad j = 1, 2, \dots, n, \quad (2)$$

where the subscript  $j$  indicates the  $j$ th joint and  $n$  is the total number of joints. In the current study equation (2) is applied to the L5-S1 joint. The error term  $\varepsilon_2$  is influenced by errors in all SIPs. Additionally, erroneous joint center estimates can influence  $\varepsilon_2$ . However, as the joint center estimates are equal in the proportional and the geometric model, influences of errors in joint center estimates should be the same for both models.

In static situations the center of pressure of the force-plate (COP) should equal the horizontal position of the body center of gravity ( $COG_{hor}$ ). Therefore,

$$COP = COG_{hor} + \varepsilon_3. \quad (3)$$

With respect to the SIPs the error term  $\varepsilon_3$  is influenced by errors in segment mass and segment relative center of gravity estimates. In dynamic situations  $\varepsilon_3$  is not only influenced by model errors but also by a real deviation between the center of pressure and the body center of gravity. In the slow lifting cycles the lowering as well as the lifting phase took 4 s and therefore these movements can be regarded as quasi-static.

In the fast lifting cycles, the real deviation between COP and  $COG_{hor}$  could be large. Then the error term  $\varepsilon_3$  is not very informative with respect to model errors. However, according to Yeadon (1990b) the external moment of the whole body ( $M_{ext}$ ) equals the rate of change of the angular momentum ( $dh/dt$ ).

Therefore, the following equation is interesting in slow as well as fast lifting movements:

$$M_{ext} = dh/dt + \varepsilon_4. \quad (4)$$

In equation (4) the external moment ( $M_{ext}$ ) of a moving multi body system is calculated as

$$M_{ext} + a \times F_{gr}. \quad (5)$$

where  $a \times F_{gr}$  is the vectorproduct between the vector from the body center of gravity to the line of action of the ground reaction force ( $a$ ) and the ground reaction force vector ( $F_{gr}$ ). Furthermore, ( $dh/dt$ ) is the rate of change of the angular momentum of the whole body (the body moment). According to Yeadon (1990b) the angular momentum ( $h$ ) of the whole body is

calculated as

$$\mathbf{h} = \mathbf{I}_b \boldsymbol{\omega}_b = \sum_{j=1}^n (\mathbf{I}_j \boldsymbol{\omega}_j + m_j \mathbf{r}_j \times \mathbf{v}_j), \quad (6)$$

where  $n$  is the number of segments,  $\mathbf{I}_b$  is the moment of inertia of the whole body and  $\boldsymbol{\omega}_b$  is the angular velocity of the whole body;  $\mathbf{I}_j$  is the moment of inertia,  $\boldsymbol{\omega}_j$  is the angular speed and  $m_j$  is the mass of segment  $j$ . Furthermore  $\mathbf{r}_j \times \mathbf{v}_j$  is the vectorproduct between the distance and the linear velocity of the center of gravity of segment  $j$  with respect to the body center of gravity.

In most human movements the actual external moment in small, meaning that the moment arm of the external moment (i.e. the shortest distance between the ground reaction force vector and the body center of gravity) will be small. Therefore, errors in the estimated location of the body center of gravity will strongly influence the calculated  $M_{\text{ext}}$ . Consequently, segment mass and segment relative center of gravity errors (and additionally, through the right-hand side of the equation, segment moment of inertia errors) can influence the error term  $\varepsilon_4$ .

#### Treatment of the data

Complete lowering/lifting cycles for all calculated parameters were normalized to mean movement time by means of a cubic spline function. From these data mean curves over subjects and curves of the standard error of the mean were calculated.

#### Statistical analysis

Differences in the SIP values resulting from the proportional and from the geometric model were tested using a Student's  $t$ -test for paired observations. Pearson coefficients of correlation between the time series of the left- and the right-hand side of the equations (excluding the error terms) were calculated. Differences between the coefficients of correlation of the proportional model and the geometric model were tested using a Wilcoxon signed rank test for paired observations.

Subsequently, for all subjects and both models, mean and absolute mean value errors were calculated. Since equation (3) is not useful for fast movements, the calculations were not performed for the fast lifting movements in this equation. Furthermore, mean force errors in equation (1) were not calculated. Because the subject stood still at the beginning and the end of each trial ( $\mathbf{F}_g + m \cdot \mathbf{g}$ ) as well as  $\sum m \cdot \mathbf{a}$  are expected to be zero, no matter what SIPs are chosen. Mean value errors in equations (2)–(4) and peak moment errors in equation (2) were tested for deviations from zero using a Student's  $t$ -test.

An analysis of variance was performed on the mean and absolute mean value errors of equations (2)–(4) and on the peak and absolute peak moment errors in equation (2). Model, technique and velocity were the independent variables. In the cases where significant effects were found, contrast tests for means were performed to further specify the effects. For all tests a  $p$ -value smaller than 0.05 was considered significant.

## RESULTS

#### Segment inertial parameter differences between models

The mean masses, relative positions of the centers of gravity and moments of inertia resulting from the proportional and the geometric model are presented in Table 2. In the last panels the differences between the models are given, which prove to be significant for nearly all segments and parameters. The impact of these differences on the inverse dynamic analysis was tested by equations (1)–(4). For both models median values and ranges of the coefficients of correlation are presented in Table 3.

*Equation (1): ground reaction force versus the sum of the segment masses times linear acceleration*

For the horizontal ( $y$ ) component of equation (1) the coeffi-

cients of correlation in the leg lifting techniques were significantly higher for the geometric model as compared to the proportional model. On the other hand, though marginally, for the vertical ( $z$ ) component, coefficients of correlation were higher for the proportional model in the fast lifting techniques.

*Equation (2): top-down versus bottom-up calculated net moments at the L5-S1 joint*

Figure 1 shows the mean L5-S1 moments resulting from the top-down and bottom-up analysis. The results of the proportional model and the geometric model are represented in the left and right-hand panels, respectively.

For the back technique the mean curves resulting from the proportional model indicate a difference between the top-down versus the bottom-up analysis, whereas this is not the case for the geometric model. Table 4 shows that this is reflected in significantly higher mean moments and peak moments when calculating top-down. In the leg technique only small but significant mean moment errors were found for the geometric model (Table 4).

The analysis of variance revealed an effect of the model on the peak moment error and an effect of the model, the technique and the interaction between model and technique on the mean moment error. A contrast test for means showed that the effect of the model on the mean and peak moment errors was significant for all lifting techniques.

Though it is relevant to know that there is an effect of the model, mean value errors are influenced by negative values. Consequently, it is not possible to conclude whether one of the models is better than the other. Therefore, an additional analysis of variance was performed on the absolute mean value errors. This was done for equations (3) and (4) as well. The analysis of variance on absolute mean and peak moment errors in equation (2) did not reveal an effect of the model or technique.

*Equation (3): center of pressure versus body center of gravity*

When the proportional model was applied in the back technique the estimated body center of gravity shifted a few centimeters in front of the center of pressure when the subjects bent forward (Fig. 2). Coefficients of correlation between COP and COG<sub>hor</sub> curves were significantly higher for the geometric model as compared to the proportional model in both the slow back and the slow leg technique (Table 3). However, the geometric model resulted in a small but significant mean position error in the slow leg technique (Table 4).

The analysis of variance on mean position errors revealed an effect of the model, the technique and the interaction between model and technique on the mean positions error in equation (3). The contrast test showed that the effect of the model was only significant for the slow back technique. The analysis of variance on absolute mean position errors did not reveal an effect of the model or technique.

*Equation (4): external moment versus body moment*

The effect of SIP errors in the proportional model and the geometric model become even more pronounced when the mean curves of the external moment and the body moment are studied (Fig. 3). Coefficients of correlation between the external moment and the body moment curves were significantly higher for the geometrical model as compared to the proportional model in all four lifting movements (Table 3). The mean moment error deviated significantly from zero in the proportional model for the fast back technique (Table 4). When the geometric model was applied, a smaller but still significant error was found in both the slow and the fast leg technique.

The analysis of variance revealed an effect of the model, the technique and the interaction between model and technique on the mean moment error. The contrast test showed that the effect of the model was significant in all four lifting movements. The analysis of variance on absolute mean moment errors did not reveal an effect of the model or technique.

Table 2. Segment interial properties estimated by the proportional model (prop) and the geometric model (geom)

Mass	Prop (kg)		Geom (kg)		Geom – prop (% prop)	
Segments	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)
Feet	1.9	(0.4)	1.4	(0.4)	27.5*	(14.5)
Lower legs	6.9	(1.0)	6.8	(1.0)	2.1	(9.1)
Upper legs	15.3	(2.3)	13.8	(3.2)	9.4*	(6.8)
Pelvis	6.2	(0.9)	11.8	(2.5)	– 91.3*	(29.1)
Trunk	25.6	(5.7)	22.7	(4.7)	11.3*	(8.2)
Upper arms	4.3	(0.9)	4.8	(1.0)	– 12.7*	(10.6)
Forearms	2.4	(0.6)	2.2	(0.6)	8.6*	(4.7)
Hands	0.8	(0.2)	0.9	(0.2)	– 13.1	(19.5)
Head	5.7	(1.0)	4.6	(0.7)	18.5*	(12.4)
Rel. center of gravity	Prop (Ratio)		Geom (Ratio)		Geom – prop (% prop)	
Segments	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)
Feet	0.50	(0.00)	0.59	(0.01)	– 17.3*	(2.5)
Lower legs	0.43	(0.01)	0.42	(0.01)	1.4	(3.7)
Upper legs	0.43	(0.00)	0.55	(0.01)	– 27.2*	(2.0)
Pelvis	0.71	(0.01)	1.05	(0.08)	– 47.5*	(11.1)
Trunk	0.46	(0.03)	0.53	(0.02)	– 15.5*	(8.2)
Upper arms	0.45	(0.01)	0.43	(0.01)	4.9*	(4.7)
Forearms	0.43	(0.00)	0.42	(0.00)	3.3*	(1.2)
Head	1.00	(0.00)	0.93	(0.09)	7.3*	(8.9)
Moment of inertia	Prop (kg m <sup>2</sup> )		Geom (kg m <sup>2</sup> )		Geom – prop (% prop)	
Segments	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)
Feet	0.007	(0.002)	0.002	(0.001)	71.0*	(26.7)
Lower legs	0.113	(0.030)	0.095	(0.029)	16.1*	(11.6)
Upper legs	0.320	(0.082)	0.150	(0.050)	53.3*	(10.8)
Pelvis	0.154	(0.035)	0.067	(0.024)	56.4*	(15.4)
Trunk	0.670	(0.265)	0.410	(0.143)	38.8*	(21.7)
Upper arms	0.039	(0.017)	0.045	(0.015)	– 16.3*	(10.8)
Forearms	0.015	(0.006)	0.012	(0.005)	23.4*	(9.7)
Hands	0.002	(0.000)	0.002	(0.001)	1.4	(39.9)
Heads	0.042	(0.009)	0.030	(0.008)	28.4*	(10.7)

Note: Segment mass is given in the upper, segment center of gravity (relative distance from proximal) in the middle and segment moment of inertia in the lower table. Mean values, standard deviations, mean differences between the models and standard deviation of the difference between the models are indicated. Significant differences are indicated by \*.

In the slow lifting movements the external moment and body moment are expected to be close to zero. The mean curves (Fig. 3) show that the body moment was indeed continuously close to zero. However, in the back technique the application of the proportional model resulted in a strong negative external moment (i.e. the ground reaction force vector pointed behind the body center of gravity) when the subjects bent forward. Furthermore, Fig. 3 shows that when the subjects were in erect position, both models yielded a positive external moment.

Mean values for the body moments were close to zero for both models ( $0.7 \pm 1.3$  Nm for the proportional model and  $0.6 \pm 1.0$  Nm for the geometric model, mean over all subjects, techniques and lifting speeds). This is expected as the subjects started and ended the lifting movements standing still in erect position. However, the mean external moment did deviate much more from zero than the body moment:  $10.7 \pm 8.7$  Nm for the proportional model and  $8.0 \pm 6.1$  Nm for the geometric model.

## DISCUSSION

Differences between the SIPs found for the proportional model and the geometric model in the current study (Table 2) are comparable to the SIP errors reported by Capozzo and Berme (1990). Since most differences between the SIPs estimated

by the proportional model and the geometric model are significant (Table 2), it is not too surprising that the type of anthropometric model used influences the error that is found in equations (1)–(4). However, it was not expected that the type of lifting movement could also have a strong influence on the error terms in the equations. This finding makes it difficult to draw conclusions about “the best model”. At least, such conclusions need to be accompanied by an explanation for the difference between the results for the back and the leg technique. Furthermore, some attention is to be paid to the effectiveness of the four different equations with respect to their potential to judge the quality of an anthropometric model.

### Proportional model versus geometric model

From the coefficients of correlation in equations (1)–(4), the geometric model seems to yield better results than the proportional model: Table (3) shows higher coefficients of correlation in the geometric model in 9 comparisons, whereas the proportional model resulted in higher coefficients of correlation in only 2 comparisons. Though this suggests that the geometric model estimates SIPs more accurately than the proportional model, it must be realized that coefficients of correlation do not reflect systematic differences between curves. Mean value errors were shown to differ between the models. However, mean value

Table 3. Coefficients of correlation between the ground reaction force and the sum of the segment masses times linear accelerations, between the bottom-up and top-down calculated net moment at the L5-S1 joint, between the center of pressure and the horizontal position of the body center of gravity and between the external moment and the time derivate of the angular momentum of the whole body

Coefficient of correlation	Equation	Technique	Prop		Geom	
			Median	Range	Median	Range
$F_{gr,y} - \sum m_j a_{j,y}$	(1)	Slow backlift	0.656	(0.428–0.754)	0.641	(0.423–0.736)
		Fast backlift	0.389	(–0.348–0.895)	0.658*	(0.089–0.891)
		Slow leglift	0.494	(0.347–0.753)	0.488	(0.376–0.755)
		Fast leglift	0.670	(0.388–0.872)	0.760*	(0.415–0.879)
$F_{gr,z} - \sum m_j a_{j,z}$	(1)	Slow backlift	0.863	(0.835–0.914)	0.855	(0.825–0.919)
		Fast backlift	0.997	(0.989–0.988)	0.995	(0.989–0.998)
		Slow leglift	0.947*	(0.862–0.975)	0.933	(0.810–0.970)
		Fast leglift	0.996*	(0.995–0.997)	0.995	(0.993–0.997)
$M_{f,L5-S1} - M_{h,L5-S1}$	(2)	Slow backlift	0.997	(0.955–0.997)	0.997*	(0.979–0.998)
		Fast backlift	0.990	(0.965–0.994)	0.991	(0.981–0.994)
		Slow leglift	0.993	(0.982–0.997)	0.991	(0.978–0.996)
		Fast leglift	0.991	(0.979–0.994)	0.991	(0.971–0.995)
COP–COG <sub>hor</sub>	(3)	Slow backlift	0.935	(0.362–0.989)	0.963	(0.875–0.993)
		Slow leglift	0.966	(0.781–0.990)	0.958*	(0.796–0.990)
$M_{ext} - dh/dt$	(4)	Slow backlift	–0.327	(–0.503–0.079)	–0.035*	(–0.406–0.452)
		Fast backlift	0.764	(0.225–0.968)	0.963*	(0.827–0.988)
		Slow leglift	0.284	(0.032–0.482)	0.310*	(0.109–0.574)
		Fast leglift	0.776	(0.139–0.911)	0.813*	(0.112–0.901)

Median values and ranges over all trials are given for both the proportional (prop) and the geometric (geom) model and for all four lifting movements.

\*Significantly higher coefficients of correlation in a model.

$F_{gr,y}$ ,  $F_{gr,z}$ : Ground reaction force, resp. horizontal and vertical direction.

$\sum m_j a_{j,y}$ ,  $\sum m_j a_{j,z}$ : Sum of segment masses times segment linear accelerations, resp. horizontal and vertical direction.

$M_{f,L5-S1}$ ,  $M_{h,L5-S1}$ : Net moment at the L5-S1 joint, resp. calculated bottom-up and top-down.

$M_{ext}$ : External moment of the ground reaction force with respect to the body center of gravity.

$dh/dt$ : time derivate of the angular momentum of the whole body.

COP: Center of Pressure.

COG<sub>hor</sub>: Horizontal position of the body center of gravity.

errors can be negative in one subject and positive in another subject. Therefore, a better way to judge the accuracy of an anthropometric model, is to look at absolute mean value errors. For instance, if the absolute mean value error would be lower in the geometric model as compared to the proportional model, this would suggest that the geometric model estimates SIPs more accurately. However, the analysis of variance did not reveal an effect of the model on the absolute mean value errors in any of the equations (2)–(4).

If one or both of the models systematic errors in the SIPs, a significant deviation from zero for the mean value error in equations (2)–(4) and for the peak moment error in equation (2) is expected. Table 4 shows that indeed some deviations from zero were found. However, these results are somewhat confusing: In 5 comparisons concerning the back technique the proportional model resulted in a significant deviation from zero. On the other hand, in 5 equations concerning the leg technique the geometric model resulted in a significant deviation from zero.

Therefore, although there are some arguments to say that the SIPs of the geometric model are better than the SIPs of the proportional model, it is quite clear that the type of movement that was analyzed, strongly influenced the results.

#### Back technique versus leg technique

It seems difficult to explain the technique-dependent difference in results for the two models. One could argue that the

actual position of the center of gravity of the trunk changes during a back technique due to the curving of the trunk and that the proportional model might better estimate the center of gravity of an erect trunk whereas the geometric model might better estimate the center of gravity of a curved trunk. In this respect one should realize that the trunk center of gravity is defined in a body position where the trunk is erect. However, this holds for both models.

Another explanation for the paradoxical results is that the effect of SIP errors of specific segments on equations (1)–(4) might be different for the back and leg technique. For instance, suppose that the proportional model estimates the trunk center of gravity too high. This could explain the difference between the COP and COG<sub>hor</sub> curves for the proportional model in the back technique (Fig. 2), because the height of the trunk center of gravity only influences COG<sub>hor</sub> when the trunk is flexed. However, one could argue that in a leg technique using a barbell, also a considerable amount of trunk flexion is reached.

#### Sensitivity analysis

As equations (1)–(4) only provide information on summed SIP errors it is difficult to obtain information about the effect of errors in specific segments. In order to shed some light on the effect of SIP errors in individual segments in the different lifting techniques, a sensitivity analysis was performed. For a subject

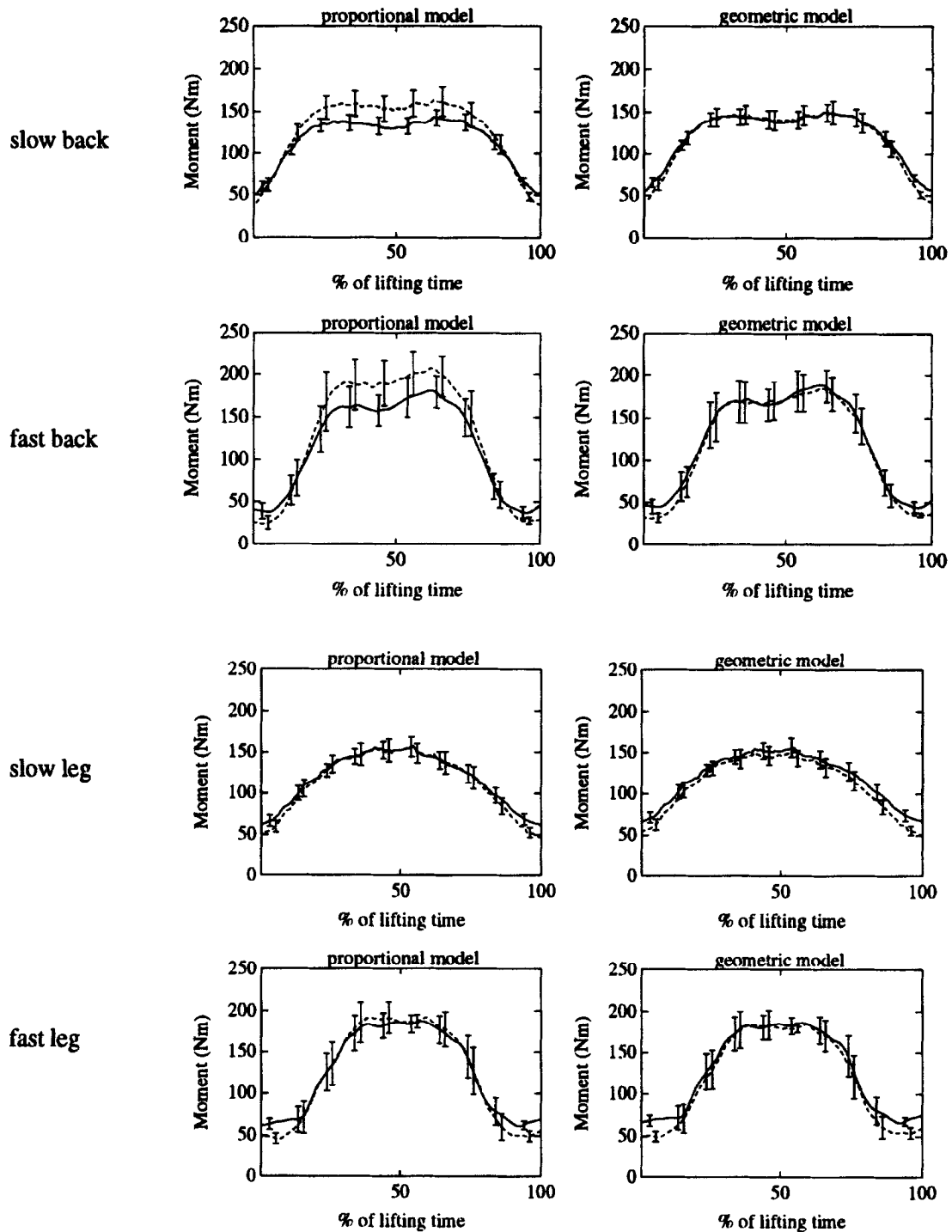


Fig. 1. Mean curves across all subjects of the bottom-up (—) and top-down (---) calculated net moments at the L5-S1 joint. The rows represent the slow back technique, the fast back technique, the slow leg technique and the fast leg technique, respectively. In the left-hand panels results of the proportional model and in the right-hand panels results of the geometric model are given. Error bars indicate 1 standard error of the mean. All curves are normalized to lifting time.

with average stature and body weight (subject M4) 10% was added to the value of each SIP of each segment as determined for the proportional model. Each time all four lifting movements were reanalyzed. The effects on the mean value errors of equations (2) and (4) and on the peak value errors of equation (2) are presented in Table 5. Some relevant effects on correlations will

be indicated in terms of changes in explained variance below. As the results of the sensitivity analysis for the slow movements were comparable to the results for the fast movements, only the effects in the fast movements are presented in Table 5.

Table 5 shows that the SIPs with the strongest influence on equations (2) and (4) are the trunk mass and center of gravity.



Table 4. Mean and standard deviation (S.D.) for mean value errors in equations (2)–(4) and peak value errors in equation (2) over all subjects for the (slow and fast) back technique and leg technique

Equation	Technique	Prop.		Geom.	
		Mean	(S.D.)	Mean	(S.D.)
(2) Mean $M_{f, L5-S1}$ —mean $M_{h, L5-S1}$ (Nm)	Slow backlift	− 12.1*	(14.3)	2.6	(8.0)
	Fast backlift	− 11.9*	(13.8)	5.0	(7.0)
	Slow leglift	3.3	(9.8)	7.8*	(8.7)
	Fast leglift	2.8	(12.4)	7.8*	(9.7)
(2) peak $M_{f, L5-S1}$ —peak $M_{h, L5-S1}$ (Nm)	Slow backlift	− 21.5*	(17.7)	0.1	(7.8)
	Fast backlift	− 31.6*	(23.2)	4.3	(10.1)
	Slow leglift	− 2.8	(14.1)	3.1	(10.9)
	Fast leglift	− 10.6	(23.3)	− 2.7	(21.3)
(3) Mean COP—mean COG <sub>hor</sub> (cm)	Slow backlift	− 1.1	(1.5)	0.7	(0.9)
	Slow leglift	0.6	(1.3)	1.2*	(1.2)
(4) Mean $M_{ext}$ —mean $dh/dt$ (Nm)	Slow backlift	− 10.8	(13.3)	3.7	(6.8)
	Fast backlift	− 13.0*	(13.9)	4.8	(6.6)
	Slow leglift	3.3	(9.8)	7.8*	(8.9)
	Fast leglift	2.4	(12.2)	7.6*	(9.5)

\*Significant deviations from zero for the error terms.

$M_{f, L5-S1}$ ,  $M_{h, L5-S1}$ : Net moment at the L5-S1 joint, resp. calculated bottom-up and top-down.

$M_{ext}$ : External moment of the ground reaction force with respect to the body center of gravity.

$dh/dt$ : Time derivate of the angular momentum of the whole body.

COP: Center of Pressure.

COG<sub>hor</sub>: Horizontal position of the body center of gravity.

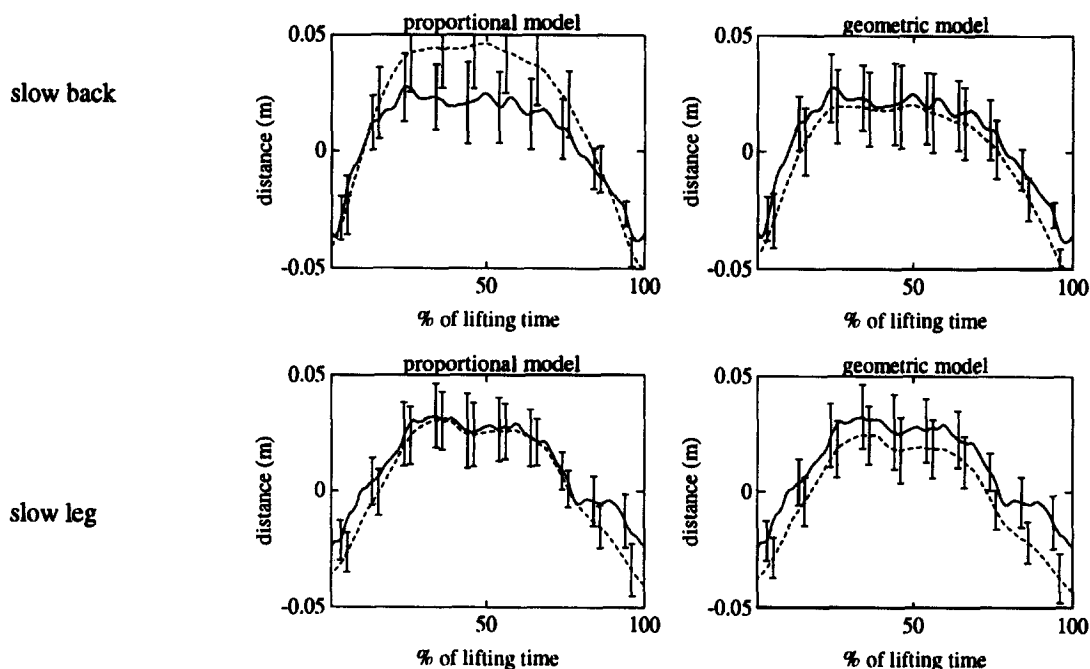


Fig. 2. Mean curves over all subjects for the center of pressure (—) and the sagittal position of the body center of gravity (---). The subjects were bending forward in the positive direction. The rows represent the slow back technique and the slow leg technique, respectively. In the left-hand panels results of the proportional model and in the right-hand panels results of the geometric model are given. Error bars indicate 1 standard error of the mean. All curves are normalized to lifting time.

Suppose that one or both of these parameters are in error. Then some effect is expected in the leg technique as well, since Table 5 shows that the effect of a change in the trunk mass or center of gravity on the mean moment error is not much lower in the leg technique, as compared to the back technique. However, in the

leg technique strong effects are also seen when masses or centers of gravity of the upper and lower legs are changed. Therefore, when evaluating the proportional model using a leg technique it could occur that errors in the trunk SIP estimates might be compensated by errors in the legs. This might explain some of

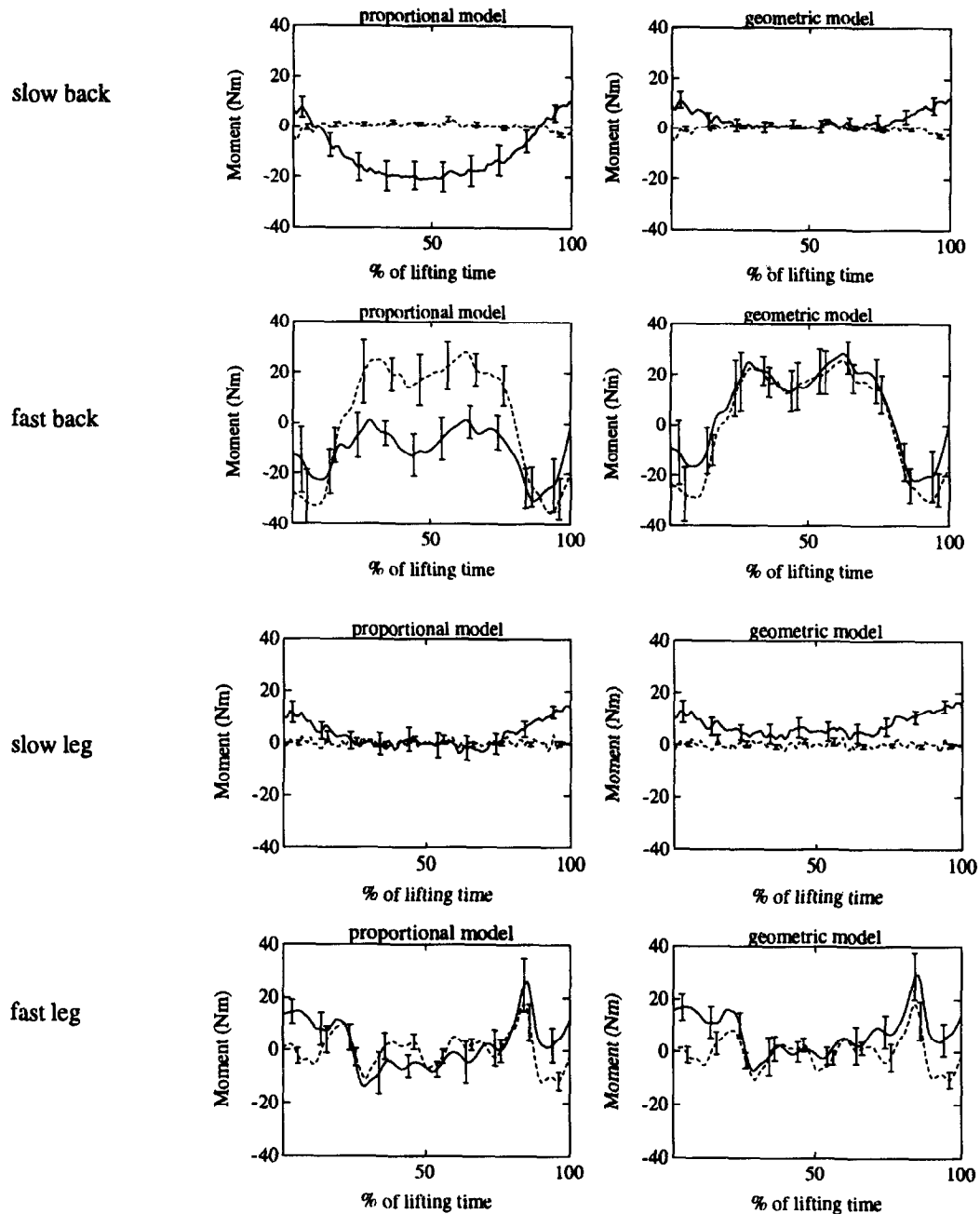


Fig. 3. Mean curves over all subjects of the external moment (—) and the body moment (---). The rows represent the slow back technique, the fast back technique, the slow leg technique and the fast leg technique, respectively. In the left-hand panels results of the proportional model and in the right-hand panels results of the geometric model are given. Error bars indicate 1 standard error of the mean. All curves are normalized to lifting time.

the difference between the results for the back and the leg technique.

#### Differences between equations

In order to test the reliability of linked segment models, some of the equations studied, have been used previously. The peak value error found in equation (2) in the current study is comparable to values reported by Looze *et al.* (1992b) in the analysis of

a back lifting technique and by Mackinnon and Winter (1993) in the analysis of gait.

With regard to equation (4), Toussaint *et al.* (1995) found a much closer relation between the body moment and the external moment. However, in their study the calculation of the external moment was based on an estimated ground reaction force (the inverse dynamic analysis started at the hands). Because the current results are based on measured ground reaction forces the results of Toussaint *et al.* (1995) cannot be compared to the results of the current study directly.

Table 5. Results of the sensitivity analysis on the fast lifting movements of subject M4 with application of the proportional anthropometric model

Sensitivity	$\Delta$ (Mean $M_{f, L5-S1}$ – mean $M_{h, L5-S1}$ ) (Nm)		$\Delta$ (Peak $M_{f, L5-S1}$ – peak $M_{h, L5-S1}$ ) (Nm)		$\Delta$ (mean $M_{ext}$ – mean $dh/dt$ ) (Nm)	
Effect of mass	Fast back	Fast leg	Fast back	Fast leg	Fast back	Fast leg
Feet	0.20	0.30	0.22	0.32	– 0.09	– 0.08
Lower legs	0.43	1.63	0.47	2.07	– 0.72	0.09
Upper legs	0.09	2.12	– 0.02	2.76	– 2.39	– 1.25
Pelvis	– 0.18	0.03	– 0.25	0.00	1.27	– 1.43
Trunk	3.56	2.92	5.18	4.13	– 0.15	– 2.04
Upper arms	1.29	1.32	1.69	1.77	0.67	0.48
Forearms	0.73	0.90	0.93	1.16	0.40	0.45
Hand	0.24	0.35	0.31	0.45	0.14	0.20
Head	2.23	1.77	3.26	2.50	1.35	0.59
Rel COG						
Feet	0.10	0.08	0.10	0.07	0.10	0.08
Lower legs	– 0.02	– 0.55	0.02	– 0.72	– 0.02	– 0.56
Upper legs	0.67	2.01	0.78	2.60	0.67	2.02
Pelvis	– 0.18	0.03	– 0.25	0.00	– 0.18	0.03
Trunk	– 3.42	– 2.81	– 5.13	– 4.02	– 3.43	– 2.82
Upper arms	– 0.01	– 0.17	0.03	– 0.12	– 0.01	– 0.17
Forearms	– 0.04	– 0.13	– 0.04	– 0.17	– 0.04	– 0.13
Hands	0.04	0.27	0.05	0.36	0.04	0.28
Head	0.57	0.64	0.54	0.69	0.57	0.65
Inertia						
Feet	0.000	– 0.000	0.000	– 0.000	0.000	– 0.000
Lower legs	– 0.000	0.001	0.003	0.000	– 0.000	– 0.001
Upper legs	0.000	– 0.007	0.101	– 0.521	0.000	– 0.007
Pelvis	0.001	– 0.000	0.210	– 0.097	0.001	– 0.000
Trunk	0.015	0.005	0.943	– 0.054	0.015	0.005
Upper arms	0.000	– 0.000	– 0.010	– 0.004	0.000	– 0.000
Forearms	0.000	– 0.000	– 0.006	– 0.002	0.000	– 0.000
Hands	– 0.001	– 0.000	– 0.001	0.005	– 0.000	– 0.000
Head	0.001	0.000	0.063	– 0.004	0.001	0.000

Each time 10% of the original value was added to one inertial parameter (segment mass in the upper, segment relative center of gravity in the middle and segment moment of inertia in the lower table). The influence of each 10% change on the mean moment error of equations (2) and (4) and on the peak moment error of equation (2) is given.

$M_{f, L5-S1}$ ,  $M_{h, L5-S1}$ : Net moment at the L5-S1 joint, resp. calculated bottom-up and top-down.

$M_{ext}$ : External moment of the ground reaction force with respect to the body center of gravity.

$dh/dt$ : time derivate of the angular momentum of the whole body.

The coefficients of correlation in the horizontal component of equation (1) in the current study are somewhat higher than the mean correlation of 0.34 reported by Looze *et al.* (1992b) but they can still be characterized as quite low. With respect to the vertical component in equation (1) coefficients of correlation of 0.43 (Freivalds *et al.*, 1984), 0.65 (Kromodihardjo and Mital, 1986) and 0.88 (Looze *et al.*, 1992b) have been reported in the literature. Those values are substantially lower than the values found in the current study (see Table 4), especially compared to the current results on the fast lifting movements. This might be due to the relatively strong filtering of force signals in the current study and in the fast lifting movements probably also to the high speed of the movements, which causes a higher signal-to-noise ratio. In the current study the highest coefficients of correlation for the vertical component of equation (1) were reached in the leg technique. This might be explained by compensation of segment mass errors in other segments because all segments accelerate in the same vertical direction.

In general, equations (1) and (2) do not seem to be very sensitive to SIP errors. In the sensitivity analysis a maximum change of the explained variance of about 1% was found for both equations. In contrast, changes in the explained variance up to 10% were found for equation (4). This difference in the sensitivity for SIP errors between equations (2) and (4) is only found for the coefficients of correlation. The mean moment error

in equation (4) is comparable to the mean moment error in equation (2) (see Table 4). In fact, the sensitivity analysis (Table 5) shows that 10% changes in segment center of gravity and moment of inertia estimates have almost the same influence on the mean moment errors of equations (2) and (4). So the specific advantage of equation (4) over equation (2) with respect to the judgement of the quality of anthropometric models is only caused by the small actual values of the external moments in activities like lifting a load. Therefore, the signal-to-noise ratio is smaller for equation (4) as compared to equation (2). This implies that the small external moments render it difficult to obtain reliable measures of this parameter, since errors due to SIP errors of an anthropometric model can easily exceed the actual values.

In conclusion, the current study provides some indication that the geometric model yields better estimates for SIPs than the proportional model. However, this is not unequivocally proved since the performance of both models was dependent on the type of lifting movement that was studied. Therefore, generalization of the current results to other types of human movement is difficult.

The relation between the external moment and the body moment is the most sensitive to SIP errors. Consequently, this relation is to be preferred when judging the overall SIP quality of an anthropometric in a full body analysis of human movement.

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## APPENDIX A

In order to calculate the volume, center of gravity and moment of inertia of a solid, bounded by a lower stadium with subscript 0 and bounded by an upper stadium with subscript 1 (see Fig. A1), Yeadon (1990a) defines the following functions:

$$F1(a, b) = 1 + (a + b)/2 + ab/3$$

$$F2(a, b) = 1/2 + (a + b)/3 + ab/4$$

$$F3(a, b) = 1/3 + (a + b)/4 + ab/5$$

$$F4(a, b) = 1 + (a + 3b)/2 + (3ab + 3b^2)/3$$

$$+ (3ab^2 + b^3)/4 + ab^3/5,$$

$$F5(a, b) = 1 + (2a + 2b)/2 + (a^2 + 4ab + b^2)/3$$

$$+ 2ab(a + b)/4 + a^2b^2/5,$$

where

$$a = (r_1 - r_0)/r_0,$$

$$b = (t_1 - t_0)/t_0,$$

with

$$r = (p - 2w)/(2\pi - 4),$$

$$t = (\pi w - p)/(2\pi - 4),$$

where  $p$  is the perimeter and  $w$  is the width of the segment at the level of a stadium. Then the volume ( $V$ ) of the solid is defined as

$$V = hr_0 [4t_0 F1(a, b) + \pi r_0 F1(a, a)],$$

where  $h$  is the total solid height.

The height of the center of gravity with respect to the lowest solid ( $z$ ) is

$$z = h^2 [4r_0 t_0 F2(a, b) + \pi r_0^2 F2(a, a)]/V.$$

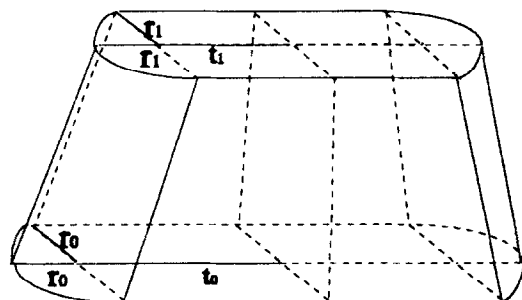


Fig. A1. A stadium solid according to Yeadon (1990a).

The principal moment of inertia about the smallest horizontal solid axis is

$$I_x^0 = Dh [4r_0 t_0^3 F4(a, b)/3 + \pi r_0^4 F4(a, a)/4] \\ + Dh^3 [4r_0 t_0 F3(a, b) + \pi r_0^2 F3(a, a)].$$

The principal moment of inertia about the largest horizontal axis is

$$I_y^0 = Dh [4r_0 t_0^3 F4(a, b)/3 + \pi r_0^2 t_0^2 F5(a, b) \\ + 8r_0^3 t_0 F4(b, a)/3 + \pi r_0^4 F4(a, a)/4] \\ + [4r_0 t_0 F3(a, b) + \pi r_0^2 F3(a, a)].$$